

Can scalar leptoquarks explain the f_{D_s} puzzle?

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(Dated: July 7, 2009)

Motivated by the disagreement between experimental and lattice QCD results on the D_s decay constant we systematically reinvestigate role of leptoquarks in charm meson decays. We consider scalar leptoquarks that transform as a weak interaction triplet, doublet, or singlet in a model independent approach, and also argue that in a particular $SU(5)$ GUT model these leptoquark states, contained in the 45-dimensional Higgs representation, could be safe against proton decay bounds. Using the current experimental measurements in τ , kaon and charm sectors, we find that scalar leptoquarks cannot naturally explain the $D_s \rightarrow \mu\nu$ and $D_s \rightarrow \tau\nu$ decay widths simultaneously. While any contributions of the triplet leptoquarks are already excluded, the singlets could only contribute significantly to the $D_s \rightarrow \tau\nu$ width. Finally, a moderate improvement of the experimental upper bound on the $D^0 \rightarrow \mu^+\mu^-$ decay width could exclude the doublet contribution to the $D_s \rightarrow \mu\nu$, while present experimental data limits its mass to be below 1.4 TeV. Possible new signatures at present and near future experiments are also briefly discussed.

PACS numbers: 13.20.Fc, 12.10.Dm, 12.15.Ff

I. INTRODUCTION

Leptoquark states are expected to exist in various extensions of the Standard Model (SM). They were first introduced in the early grand unification theories (GUTs) in the seventies [1, 2]. Scalar leptoquarks are expected to exist at TeV scale in extended technicolor models as well as in models of quark and lepton compositeness. Scalar quarks in supersymmetric models with R-parity violation (RPV) may also have leptoquark-type Yukawa couplings [32].

Recently, discrepancies between the experimental measurements of leptonic decay modes of D_s mesons [3–6] and the lattice results for the relevant f_{D_s} decay constant [7–9] have stimulated many analyses. One intriguing indication is that the central measured and predicted values for the f_{D_s} differ by more than 10% with a combined significance of $D_s \rightarrow \tau\nu$ and $D_s \rightarrow \mu\nu$ channels of roughly 2.3σ [10], while the corresponding values for f_D are in perfect agreement. In Ref. [11] the idea of scalar leptoquarks has been revived to explain the missing decay widths. Some implications of this suggestion have been further explored using semileptonic [12, 13] and rare charm decays [14].

Generally, leptoquarks which also couple to diquarks mediate fast proton decay and are therefore required to be much above the electroweak scale [15], making them uninteresting for other low energy phenomena. “Genuine” leptoquarks on the other hand, couple only to pairs

of quarks and leptons, and may thus be inert with respect to proton decay. In such cases, proton decay bounds would not apply and leptoquarks may produce signatures in other low-energy phenomena. In this article we set out to study whether scalar leptoquarks can naturally account for the f_{D_s} puzzle and at the same time comply with all other measured flavor observables.

We consider all possible renormalizable leptoquark interactions with SM matter fields consistent with the SM gauge symmetry. One can construct such dimension-four operators using leptoquarks which are either singlets, doublets or triplets under the $SU(2)_L$. If we furthermore require that such leptoquarks contribute to leptonic decays of charged mesons at tree level, we are left with three possible representation assignments for the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge groups: $(\mathbf{3}, \mathbf{3}, -1/3)$, $(\bar{\mathbf{3}}, \mathbf{2}, -7/6)$ and $(\mathbf{3}, \mathbf{1}, -1/3)$. Only the weak doublet leptoquark is “genuine” in the above sense. However, using a concrete $SU(5)$ GUT model where the relevant leptoquarks are embedded into the 45-dimensional Higgs representation ($\mathbf{45}_H$), we demonstrate how leptoquark couplings to matter can arise and in particular, how the dangerous couplings to diquarks – both direct and indirect [15] – can be avoided.

II. GENERAL CONSIDERATIONS

In our analysis we will assume the mass eigenstates within a leptoquark weak multiplet to be nearly degenerate. While large mass splittings within a weak multiplet may be considered unnatural, more importantly, they are also tightly constrained by electroweak precision observable T [16]. Consequently, one generically gets correlations between semileptonic charged currents

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and (lepton) flavor violating neutral currents, which represent important constraints on any leptoquark scenario trying to resolve the D_s leptonic widths puzzle. Also, we focus on observables mediated by the relevant leptoquark couplings at tree level since these already involve processes forbidden in the SM at tree level, i.e., flavor changing neutral currents (FCNCs) and lepton flavor violation (LFV) processes. Finally, since the present f_{D_s} deviation is of mild significance, we require all the measured constraints to be satisfied within one standard deviation (at 68 % C.L.) except upper bounds, for which we use published 90 % C.L. limits. We consider a leptoquark explanation of the f_{D_s} discrepancy as natural, if both D_s and D leptonic decay widths can be obtained close to their measured central values.

After the electroweak (EW) symmetry breaking, quarks and leptons acquire their masses from their respective Yukawa interactions. These induce in the physical (mass eigen-)basis a CKM and PMNS rotations between the upper and the lower components of the fermion doublets. Consequently, it is impossible to completely isolate leptoquark mediated charged current interactions to a particular quark or lepton generation in the left-handed sector *irrespective of the initial form of the leptoquark couplings to SM matter fields, unless there is some special alignment with the right-handed quark sector*. To see this, we denote as $X^{(i)}$ a 3×3 arbitrary Yukawa matrix in the weak basis, and write down flavor structure of interaction of the quark and lepton doublet parts

$$\overline{Q}_q^w X^{q\ell} = (\overline{u}_q^w \quad \overline{d}_q^w) X^{q\ell} = (\overline{u}_q \quad \overline{d}_q)(U^\dagger X)^{q\ell}, \quad (1a)$$

$$X'^{q\ell} L_\ell^w = X'^{q\ell} (\nu_\ell^w \quad e_\ell^w)^T = (X' E)^{q\ell} (\nu_\ell \quad e_\ell)^T, \quad (1b)$$

where fields with w superscript are in the weak basis, whereas $d' = V_{CKM} d$ and $\nu' = V_{PMNS} \nu$. The unitary matrices U, D, E , and N rotate the fields from mass to weak basis and are unphysical *per se*, so we absorb them in redefinition of the couplings (e.g. $Y_{LQ} \equiv U^\dagger X$ on the quark and $Y'_{LQ} \equiv X' E$ on the lepton side) and consider them as free parameters. In what follows, we will use the convention where all the remaining rotations are assigned to down-type quark ($V_{CKM} = U^\dagger D$) and neutrino ($V_{PMNS} = E^\dagger N$) sectors, and the quark mass-eigenstates are defined as

$$(Q_1, Q_2, Q_3) = \begin{pmatrix} u & c & t \\ d' & s' & b' \end{pmatrix}, \quad (d' \ s' \ b') = (d \ s \ b) V_{CKM}^T.$$

It is obvious in this notation that, even if the Y_{LQ} matrix had all rows, except for the q -th one, set to zero, which would correspond to leptoquark coupling *only* to u_q , one would still get non-zero couplings to all three left-handed down-quarks. Same rationale holds true for the lepton sector due to V_{PMNS} , but since the neutrino flavors are not tagged in present experiments, the respective decay widths are summed over all neutrino flavors. Whenever a mass-eigenstate antineutrino $\bar{\nu}_i$ is produced in a reaction, its amplitude includes, according to Eq. (1), a factor of $\sum_j Y_{LQ}^{iqj} V_{PMNS}^{ji}$ for leptoquark interaction, or V_{PMNS}^{li} if

the neutrino was produced in $W\ell\nu$ vertex. In any case, when one sums the rates for all neutrino species

$$\sum_{i=1,2,3} |\mathcal{A}_i|^2 \sim \sum_{i=1,2,3} V_{PMNS}^{ji} V_{PMNS}^{li*} = \delta^{jl},$$

it becomes evident that in the summed rate all the neutrino indices are replaced by the lepton flavors. This is equivalent to the absence of mixing in the lepton doublets.

The above considerations are more general and similar in spirit to the ones recently discussed in Ref. [17] for the case of $K - \bar{K}$ and $D - \bar{D}$ mixing. In fact, any new physics coupling to SM fermionic weak doublets exhibits similar kind of correlations, and contributions to charged current transitions cannot be isolated to a particular quark generation.

Another important particularity of the f_{D_s} puzzle is that it affects a Cabibbo favored $c \rightarrow s$ transition. Consequently, the hierarchy of correlations with other processes is largely determined by the CKM mixing hierarchy. In particular, the mixing of the third generation with the first two is much smaller than the mixing of the first two generations among each other. Therefore, for our purposes, it is often a good approximation, to completely neglect effects of the third generation in the quark sector. Then we can parameterize a generic leptoquark coupling in the weak basis using a common (real) prefactor and a rotation angle

$$Y_{LQ}^{q\ell} = y_{LQ}^\ell (\sin \phi, \cos \phi).$$

In addition, the only CKM rotation is due to the Cabibbo angle and there is no SM CP violating phase ($d' = \cos \theta_c d + \sin \theta_c s$, $s' = -\sin \theta_c d + \cos \theta_c s$ and $V_{us} = -V_{cd} = \sin \theta_c = 0.225$). The absence of SM phases is not critical for our purposes, since we only consider CP conserving quantities, and since the relevant SM amplitudes in our considered processes have approximately the same weak phase even in the full three generation case. The leptoquark couplings themselves, however, could in principle have arbitrary new phases. These could be important in processes with two or more interfering amplitudes contributing, at least one of those being due to the leptoquarks. We deal with this possibility on a case by case basis. Finally, in all the scenarios considered we have checked explicitly that the two generation approximation is valid by performing numerical leptoquark parameter scans including the full CKM structure and a full set of possible leptoquark couplings with arbitrary phases. In this case, (semi)leptonic B decays $B \rightarrow \tau \nu$ and especially $B \rightarrow D \tau \nu$ can be used to put additional constraints on the leptoquark parameters relevant to the $D_s \rightarrow \tau \nu$ width. Numerically however, these constraints turn out not be competitive with the others due to presently limited experimental precision.

III. TRIPLET LEPTOQUARK (3, 3, -1/3)

The triplet leptoquark can in principle couple to diquarks and thus destabilize the proton, so one has to check in an underlying model if that is indeed the case. The allowed leptoquark interaction Lagrangian consists of a single term

$$\mathcal{L}_3 = Y_3^{ij} \bar{Q}_i^c \tau_2 \boldsymbol{\tau} \cdot \boldsymbol{\Delta}_3^* L_j + \text{h.c.}, \quad (2)$$

where $\bar{Q}^c = -Q^T C^{-1}$, $C = i\gamma^2 \gamma^0$ and $\boldsymbol{\tau}$ are the Pauli matrices. The 3×3 coupling matrix Y_3 is arbitrary in the bottom-up approach. On the other hand, its entries may be related to other parameters in an UV embedding of the effective theory. In the concrete $SU(5)$ model analyzed in the Appendix A, the above couplings are due to the contraction of **10** and $\bar{\mathbf{5}}$ with $\mathbf{45}_H^*$ – also responsible for giving masses to the down quarks and charged leptons. A different contraction of **10** and **10** with $\mathbf{45}_H$ couples the triplet to diquarks. The latter term can be consistently set to zero in the supersymmetric version of the model, thus sufficiently suppressing proton decay.

As already mentioned in Ref. [11], the triplet leptoquarks cannot by themselves account for deviations in $D_s \rightarrow \ell \nu$ for both τ and μ in the final state due to constraints coming from LFV tau decays, such as $\tau \rightarrow \eta^{(\prime)} \mu$ and $\tau \rightarrow \phi \mu$. Numerically, the $\tau \rightarrow \eta \mu$ decay turns out to be most constraining. The triplet leptoquark contribution can be written as

$$\Gamma_{\tau \rightarrow \eta \mu}^{(3)} = \frac{\left| \sum_{q=u,d,s} \chi_q \tilde{Y}_3^{q\tau} \tilde{Y}_3^{q\mu*} f_\eta^q \right|^2}{512\pi m_{\Delta_3}^4} m_\tau^3 \left[1 - \left(\frac{m_\eta}{m_\tau} \right)^2 \right]^{3/2},$$

where we have neglected the muon mass. The weight $\chi_q = 1$ is for $q = u$ and 2 for $q = d, s$ comes from an additional $\sqrt{2}$ factor in the interaction terms with $\Delta_3(t_3 = \pm 1)$ states. Decay constants of η meson f_η^q are defined as in [18]. As explained in the previous section, the couplings \tilde{Y}_3 contain an additional V_{CKM} rotation for the down-type quarks

$$\tilde{Y}_3^{q\ell} \equiv \begin{cases} Y_3^{q\ell} & ; q = u, c, t, \\ (V_{CKM}^T Y_3)^{q\ell} & ; q = d, s, b. \end{cases}$$

Thus, the upper bound on $\tau \rightarrow \eta \mu$ decay width directly constrains the products $\tilde{Y}_3^{s\tau} \tilde{Y}_3^{s\mu*}$, $\tilde{Y}_3^{d\tau} \tilde{Y}_3^{d\mu*}$ and $Y_3^{u\tau} Y_3^{u\mu*} (= (\cos \theta_c \tilde{Y}_3^{d\tau} + \sin \theta_c \tilde{Y}_3^{s\tau})(\cos \theta_c \tilde{Y}_3^{d\mu*} + \sin \theta_c \tilde{Y}_3^{s\mu*})$ in the two-generations approximation). On the other hand, the relative contributions to the D_s leptonic widths (see Eq. (3)) in the two generations approximation are $\tilde{Y}_3^{s\mu}(\tilde{Y}_3^{s\mu*} - \tan \theta_c \tilde{Y}_3^{d\mu*})$ for the muon channel and $\tilde{Y}_3^{s\tau}(\tilde{Y}_3^{s\tau*} - \tan \theta_c \tilde{Y}_3^{d\tau*})$ for the tau channel. Both cannot be sizable and at the same time agree with the bounds that come from the $\tau \rightarrow \eta \mu$ decay width [33]. In scenario of triplet leptoquarks therefore, *one of the measured leptonic channels $D_s \rightarrow \ell \nu$ would necessarily have to be a measurement artifact.* We will consider both possibilities separately.

If the leptoquarks have sizable coupling $\tilde{Y}_3^{s\tau}$ (implying $\tilde{Y}_3^{s\mu} \sim 0$ by the $\tau \rightarrow \eta \mu$ decay width) we can obtain a non-zero contribution to the $D_s \rightarrow \tau \nu$ decay width due to the interference term between the SM and the leptoquark amplitude in

$$\Gamma_{D_s \rightarrow \tau \nu}^{(3)} = \Gamma_{D_s \rightarrow \tau \nu}^{SM} \left| 1 + \frac{\delta_3^\tau}{4\sqrt{2}G_F} \right|^2,$$

where the SM width is

$$\Gamma_{D_s \rightarrow \tau \nu}^{SM} = \frac{G_F^2 m_\tau^2 |V_{cs}|^2 f_{D_s}^2 m_{D_s}}{8\pi} \left[1 - \left(\frac{m_\tau}{m_{D_s}} \right)^2 \right]^2,$$

and the relative contribution of the triplet leptoquarks reads

$$\delta_3^\tau \equiv \frac{Y_3^{c\tau*} \tilde{Y}_3^{s\tau}}{V_{cs} m_{\Delta_3}^2}. \quad (3)$$

An important observation is that the leptoquarks in this scenario contribute to the same effective operator as the SM and thus exhibit the same helicity suppression. In the two generations approximation, the relative triplet contribution simplifies to $\delta_3^\tau = (y_3^\tau)^2 \cos \phi \sin \phi (\tan \theta_c + \cot \phi) / m_{\Delta_3}^2$. Reproducing the measured branching ratio $Br(D_s \rightarrow \tau \nu) = 0.0561(44)$ [10] while using the most precise lattice input $f_{D_s} = 241(3)$ MeV [7] would require $\sqrt{\delta_3^\tau} \approx 0.002 \text{ GeV}^{-1}$. On the other hand the $Y_3^{u\tau}$ coupling of leptoquarks is constrained by precise measurement of the lepton flavor universality ratio $\pi_{\mu/\tau} \equiv Br(\tau \rightarrow \pi \nu) / Br(\pi \rightarrow \mu \nu) = 0.1092(7)$ [19, 20]. Leptoquarks contribute to semileptonic tau decays in the form

$$\Gamma_{\tau \rightarrow \pi \nu}^{(3)} = \Gamma_{\tau \rightarrow \pi \nu}^{SM} \times \left| 1 + \frac{1}{4\sqrt{2}G_F} \left[\frac{Y_3^{u\tau*} \tilde{Y}_3^{d\tau}}{V_{ud} m_{\Delta_3}^2} \right] \right|^2, \quad (4)$$

where

$$\Gamma_{\tau \rightarrow \pi \nu}^{SM} = \frac{G_F^2 m_\tau^3 |V_{ud}|^2 f_\pi^2}{16\pi} \left[1 - \left(\frac{m_\pi}{m_\tau} \right)^2 \right]^2.$$

In the two generations approximation the term in square brackets can be written as $(y_3^\tau)^2 \sin \phi \cos \phi (-\tan \theta_c + \tan \phi) / m_{\Delta_3}^2$. To exactly satisfy both the $\pi_{\mu/\tau}$ value and explain leptonic $D_s \rightarrow \tau \nu$ excess we need either

- (a) $\tan \phi \approx \tan \theta_c$, i.e., leptoquarks couple only to s but not to d quark ($\tilde{Y}_3^{d\tau} \approx 0$), or
- (b) $\sin \phi \approx 0$, i.e., leptoquarks couple only to c but not to u quark ($Y_3^{u\tau} = 0$).

However, in the limit (a) one must have a sizable coupling $Y_3^{u\tau}$ due to CKM rotation which results in relative contribution of size δ_3^τ to the Cabibbo suppressed semileptonic tau decays $\tau \rightarrow K \nu$ (of the form (4) with appropriate flavor replacement $d \rightarrow s$ and $\pi \rightarrow K$). These are measured in agreement with the SM at the

3 % level [34] (in particular the ratio $K_{\mu/\tau} \equiv Br(\tau \rightarrow K\nu)/Br(K \rightarrow \mu\nu) = 0.0109(4)$ [19, 20]) [35].

In the other limit, (b), one must have sizable coupling $\tilde{Y}_3^{d\tau}$ and thus gets a relative contribution scaling as δ_3^τ to the $D \rightarrow \tau\nu$ decay width. Currently only an upper bound exists for this channel $Br(D \rightarrow \tau\nu) < 1.2 \times 10^{-3}$ at 90 % C.L. [4]. Even more importantly, one gets a non-vanishing contribution to the rare $K^+ \rightarrow \pi^+\nu\bar{\nu}$ decay. Since the triplet leptoquark contributes with the same effective operator as the SM, its contribution can be obtained by simply replacing the $\lambda_t X_t$ product in the master formula of [21] with

$$\lambda_t X_t \rightarrow \lambda_t X_t + \frac{\sqrt{2}\pi \sum \tilde{Y}_3^{s\ell} \tilde{Y}_3^{d\ell*}}{G_F \alpha_{em} \sin \theta_W m_{\Delta_3}^2}.$$

This process is measured to have $Br(K^+ \rightarrow \pi^+\nu\bar{\nu}) = (17.3_{-10.5}^{+11.5}) \times 10^{-11}$ [22, 23]. It constraints the sum of leptoquark coupling combinations $\tilde{Y}_3^{s\ell} \tilde{Y}_3^{d\ell*} = (y_3^\ell)^2 \cos \theta_c \cos \phi (\tan \phi - \tan \theta_c) (1 + \tan \theta_c \tan \phi)$ and fixes very accurately $\tan \phi = \tan \theta_c$ or $\tan \phi = -\cot \theta_c$. The constraint applies to all lepton flavors since it is inclusive with respect to neutrino flavor [36]. The combined impact of all these constraints on the triplet leptoquark contribution to $D_s \rightarrow \tau\nu$ is shown on the first plot in Fig. 1. One observes that the combination of the strong bounds coming from $K^+ \rightarrow \pi^+\nu\bar{\nu}$ combined with $Br(\tau \rightarrow K\nu)/Br(K \rightarrow \mu\nu)$ completely excludes the triplet leptoquarks from explaining the $D_s \rightarrow \tau\nu$ excess.

In the opposite scenario where the leptoquarks couple to muons the situation is similar to the tau case with two differences: (1) the $D \rightarrow \mu\nu$ decay width has already been measured and the $Br(D \rightarrow \mu\nu) = 3.8(4) \times 10^{-4}$ [4] agrees perfectly with the SM prediction using the most precise lattice QCD value of $f_D = 208(4)$ MeV [7]; (2) an additional constraint comes from the FCNC decay $K_L \rightarrow \mu^+\mu^-$ as it receives contributions from leptoquarks of the form

$$\Gamma_{K_L \rightarrow \mu^+\mu^-}^{(3)} = \frac{f_K^2 m_K m_\mu^2 \sqrt{1 - \frac{4m_\mu^2}{m_K^2}} |\tilde{Y}_3^{s\mu} \tilde{Y}_3^{d\mu*}|^2}{64\pi m_{\Delta_3}^4}.$$

The requirement that such leptoquark contributions do not exceed presently measured $Br(K_L \rightarrow \mu^+\mu^-) = 6.84(11) \times 10^{-9}$ [19] produces a bound equivalent to the existing one coming from $K^+ \rightarrow \pi^+\nu\bar{\nu}$. Combining these two additional constraints with the rest also clearly disfavors a triplet leptoquark explanation of the $D_s \rightarrow \mu\nu$ excess, as shown on the bottom plot in Fig. 1.

IV. DOUBLET LEPTOQUARK ($\bar{3}, 2, -7/6$)

The doublet leptoquarks are innocuous as far as proton decay is concerned. The allowed dimension four interactions in this case are

$$\mathcal{L}_2 = Y_{2L}^{ij} \bar{Q}_i i\tau_2 \Delta_2^* e_j + Y_{2R}^{ij} \bar{u}_i \Delta_2^\dagger L_j + \text{h.c.} \quad (5)$$

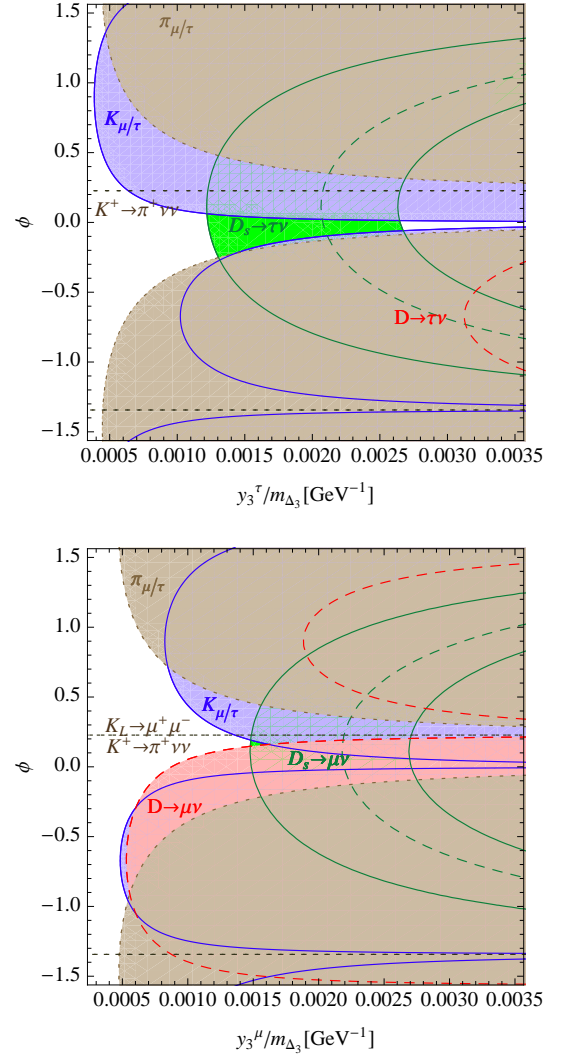


Figure 1: Combined bounds on the triplet leptoquark parameters in the two-generation limit in the tau (upper plot) and muon (lower plot) sectors. All bands represent 68% C.L. exclusion intervals, except the upper bound on $D \rightarrow \tau\nu$ which is taken at 90% C.L.. The $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $K_L \rightarrow \mu^+\mu^-$ constraints can only be satisfied on the two horizontal dashed lines. Within the green bands, the $D_s \rightarrow \ell\nu$ excess can be accounted for.

In the particular $SU(5)$ model, the term proportional to Y_{2R} stems from the contraction of $\mathbf{10}$ and $\bar{\mathbf{5}}$ with $\mathbf{45}_H^*$ while the Y_{2L} term is due to $\mathbf{10}$ and $\mathbf{10}$ being contracted with $\mathbf{45}_H$.

In this scenario the same states couple left-handed quarks to right-handed leptons and vice versa. Consequently, only the product of both couplings can contribute to the D_s leptonic widths through the interfer-

ence with the SM

$$\Gamma_{D_s \rightarrow \ell \nu}^{(2)} = \Gamma_{D_s \rightarrow \ell \nu}^{SM} \left| 1 - \frac{\delta_2^\ell}{4\sqrt{2}G_F} \right|^2, \quad (6)$$

$$\delta_2^\ell \equiv \frac{m_{D_s}^2}{m_\ell(m_c + m_s)} \frac{Y_{2R}^{c\ell*} \tilde{Y}_{2L}^{s\ell}}{V_{cs}^* m_{\Delta_2}^2}.$$

Again, the couplings of left-handed down- and up-type quarks are misaligned

$$\tilde{Y}_{2L}^{q\ell} \equiv (V_{CKM}^\dagger Y_{2L})^{q\ell} \text{ for } q = d, s, b.$$

Note that the doublet leptoquark contribution exhibits no helicity suppression. Thus, explaining both muon and tau leptonic partial widths of D_s requires vastly different leptoquark couplings. On the other hand, now one also has to take into account the strict bound coming from the decay $D^0 \rightarrow \mu^+ \mu^-$. In the doublet leptoquark model, this mode receives potential contributions from several coupling combinations

$$\Gamma_{D^0 \rightarrow \mu^- \mu^+} = \frac{f_D^2 m_D^3}{512\pi} \sqrt{1 - \frac{4m_\mu^2}{m_D^2}} \left[\left(1 - \frac{4m_\mu^2}{m_D^2} \right) A + B \right], \quad (7)$$

where A and B contain the couplings of doublet leptoquarks

$$A = \frac{m_D^2}{m_{\Delta_2}^4 (m_c + m_u)^2} |Y_{2R}^{u\mu} Y_{2L}^{c\mu*} - Y_{2R}^{c\mu*} Y_{2L}^{u\mu}|^2,$$

$$B = \frac{1}{m_{\Delta_2}^4} \left| \frac{m_D}{m_c + m_u} (Y_{2R}^{u\mu} Y_{2L}^{c\mu*} + Y_{2R}^{c\mu*} Y_{2L}^{u\mu}) + \frac{2m_\mu}{m_D} (Y_{2R}^{c\mu*} Y_{2R}^{u\mu} + Y_{2L}^{c\mu*} Y_{2L}^{u\mu}) \right|^2.$$

Two combinations involve $Y_{2R}^{u\mu}$ which is in conjunction with $\tilde{Y}_{2L}^{s\mu}$ and $\tilde{Y}_{2L}^{s\tau}$ constrained through precision kaon and tau lepton flavor universality tests similarly as in the triplet scenario. In addition, this coupling does not contribute to the $D_s \rightarrow \mu\nu$ width (6). The remaining two combinations can be rewritten by using the Cabibbo rotation in terms of $Y_{2R}^{c\mu} Y_{2L}^{u\mu*} = Y_{2R}^{c\mu} (\cos\theta_c \tilde{Y}_{2L}^{d\mu*} + \sin\theta_c \tilde{Y}_{2L}^{s\mu*})$ and $Y_{2L}^{c\mu} Y_{2L}^{u\mu*} = (\cos\theta_c \tilde{Y}_{2L}^{s\mu} - \sin\theta_c \tilde{Y}_{2L}^{d\mu}) (\cos\theta_c \tilde{Y}_{2L}^{d\mu*} + \sin\theta_c \tilde{Y}_{2L}^{s\mu*})$. The $D \rightarrow \mu\nu$ width measurement constrains directly the size of $\tilde{Y}_{2L}^{d\mu}$. In absence of this coupling, $D^0 \rightarrow \mu^+ \mu^-$ would receive dominant contributions from just two non-interfering leptoquark amplitudes

$$A \approx \frac{m_D^2}{m_{\Delta_2}^4 (m_c + m_u)^2} |Y_{2R}^{c\mu} \tilde{Y}_{2L}^{s\mu*} \sin\theta_c|^2,$$

$$B \approx \frac{\sin^2\theta_c}{m_{\Delta_2}^4} \left| \frac{m_D}{m_c + m_u} Y_{2R}^{c\mu*} \tilde{Y}_{2L}^{s\mu} + \frac{2m_\mu}{m_D} (\tilde{Y}_{2L}^{s\mu})^2 \cos\theta_c \right|^2,$$

where A can be related to $D_s \rightarrow \mu\nu$ decay width contribution (it is proportional to $\sin^2\theta_c \cos^2\theta_c |\delta_2^\mu|^2$). The

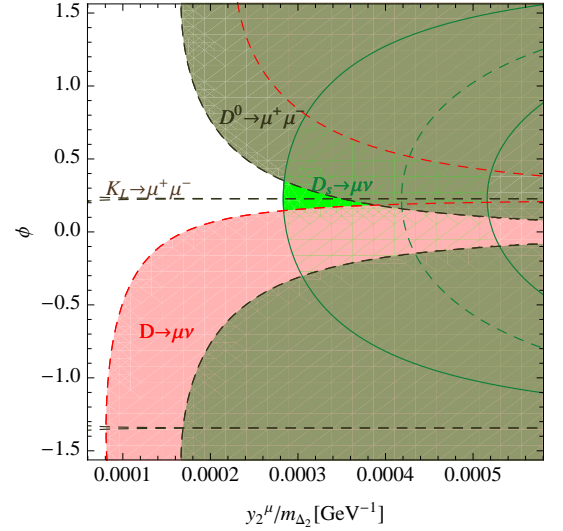


Figure 2: Combined $D^0 \rightarrow \mu^+ \mu^-$ and $D \rightarrow \mu\nu$ bounds on the doublet leptoquark parameters in the two-generation limit in muon sector as explained in the text. $D \rightarrow \mu\nu$ band represent 68% C.L. exclusion interval, while the upper bound on $D^0 \rightarrow \mu^+ \mu^-$ is taken at 90% C.L.. The $K_L \rightarrow \mu^+ \mu^-$ constraint can only be satisfied on the two horizontal dashed lines. Within the green band, the $D_s \rightarrow \ell\nu$ excess can be accounted for.

term proportional to A alone yields $Br(D^0 \rightarrow \mu^+ \mu^-) \approx 8.3 \times 10^{-7}$ for the central value of $D_s \rightarrow \mu\nu$ decay width. Recently, an improved experimental limit of $Br(D^0 \rightarrow \mu^+ \mu^-) < 4.3 \times 10^{-7}$ at 90% C.L. was put forward by CDF [24]. It is evident that this introduces some tension between explaining the D_s excess and not spoiling the agreement in the D case. Due to the moderate significance of the D_s discrepancy, this tension is not yet conclusive as can be seen on Fig. 2, where we plot the combined constraints in the ϕ - y_2^μ plane. Fig. 2 is generated in the following way. We first parameterize $\tilde{Y}_{2L}^{s\mu} = y_{2L}^\mu \cos\phi$ and $\tilde{Y}_{2L}^{d\mu} = y_{2L}^\mu \sin\phi$. We then set $Y_{2R}^{u\mu} = 0$ while $Y_{2R}^{c\mu} = y_{2R}^\mu$. Finally, we vary y_{2L}^μ and y_{2R}^μ while keeping the product $y_2^\mu = \sqrt{y_{2L}^\mu y_{2R}^\mu}$ fixed and use the best fit value to determine the allowed region. We include the $Br(K_L \rightarrow \mu^+ \mu^-)$ constraint which is also relevant in this case

$$\Gamma_{K_L \rightarrow \mu^+ \mu^-}^{(2)} = \frac{|\tilde{Y}_{2L}^{s\mu} \tilde{Y}_{2L}^{d\mu*}|^2 f_K^2 m_K^5 \sqrt{1 - \frac{4m_\mu^2}{m_K^2}} \left(1 - \frac{2m_\mu^2}{m_K^2} \right)}{256\pi m_{\Delta_2}^4 (m_d + m_s)^2}.$$

While both central values for D and D_s leptonic widths clearly cannot be reproduced by the doublet leptoquark contribution, a future improvement of the bound on $D^0 \rightarrow \mu^+ \mu^-$ is clearly sought after to reach a definite conclusion on this scenario.

Opposed to the triplet leptoquark case, the verdict on the $D_s \rightarrow \tau\nu$ contribution of the doublet leptoquark is still far from conclusive. Firstly, because the corresponding D leptonic mode has not been measured. Sec-

only, because there are presently no strong experimental bounds on FCNCs in the up quark sector involving only tau leptons or only neutrinos (doublet leptoquark does not contribute to $s \rightarrow d\nu\bar{\nu}$ transitions). We note in passing that, provided the doublet leptoquarks are to explain both tau and muon final state excesses, there is an important bound coming from the aforementioned $\tau \rightarrow \eta^{(\prime)\mu}$ decays. In this scenario they constrain the following combination of parameters $\delta_2^{LFV} = |Y_{2L}^{s\tau} Y_{2L}^{*\mu}|/m_{\Delta_2}^2$ appearing in

$$\Gamma_{\tau \rightarrow \eta\mu}^{(2s)} = \frac{|\tilde{Y}_{2L}^{s\tau} \tilde{Y}_{2L}^{*\mu}|^2 m_\tau (f_\eta^s)^2 m_\eta^4 \left(1 - \frac{m_\eta^2}{m_\tau^2}\right)^2}{128\pi m_s^2 m_{\Delta_2}^4},$$

where we have only considered the s -quark contribution and have neglected the muon mass. On the other hand, explanation of D_s leptonic excesses requires nonzero values for $\delta_2^{\mu,\tau}$. Finally, in order not to spoil perturbative treatment of the couplings, none of the couplings should exceed a value of roughly $|Y_{2L,R}^{ij}| < \sqrt{4\pi}$. Then, one can combine the above inequalities to yield a robust *upper bound on the doublet leptoquark mass*:

$$m_{\Delta_2} < \sqrt{4\pi \delta_2^{LFV} / |\delta_2^\mu \delta_2^\tau|}.$$

Taking the present $Br(\tau \rightarrow \eta\mu) < 6.5 \times 10^{-8}$ at 90 % C.L. bound [19] and the central values for the two leptonic decay widths, one obtains a value of roughly 1.4 TeV, which is certainly within the LHC reach [25].

V. SINGLET LEPTOQUARK (3, 1, -1/3)

This state was originally proposed to explain the D_s leptonic width puzzle in Ref. [11]. On the other hand, singlet leptoquarks are notorious for their mediation of proton decay. However, as in the case of the leptoquark triplet, one can demonstrate in concrete $SU(5)$ embedding (see Appendix A) that the dangerous couplings do not necessarily appear. Similarly to triplets, weak singlets can also couple to pairs of SM matter weak doublets. However, now also couplings to pairs of singlets are possible resulting in the dimension four interaction Lagrangian with two terms

$$\mathcal{L}_1 = Y_{1L}^{ij} \bar{Q}_i^c \tau_2 \Delta_1^* L_j + Y_{1R}^{ij} \bar{u}_i^c \Delta_1^* e_j + \text{h.c.} \quad (8)$$

In this generic effective theory description clear correlations among different charged and neutral current flavor observables, present in the triplet case, are somewhat diluted by the presence of the second interaction term which modifies the singlet leptoquark contribution to the D_s leptonic width

$$\begin{aligned} \frac{\Gamma_{D_s \rightarrow \tau\nu}^{(1)}}{\Gamma_{D_s \rightarrow \tau\nu}^{SM}} &= \left| 1 + \frac{1}{4\sqrt{2}G_F m_{\Delta_1}^2} \right. \\ &\times \left. \left\{ \left[\frac{Y_{1L}^{c\tau*} \tilde{Y}_{1L}^{s\tau}}{V_{cs}} \right] - \frac{m_{D_s}^2 (Y_{1R}^{c\tau*} \tilde{Y}_{1L}^{s\tau})}{V_{cs}^* m_\tau (m_c + m_s)} \right\} \right|^2, \end{aligned} \quad (9)$$

where $\tilde{Y}^{q\ell}$ are defined as in the triplet leptoquark scenario. The second term in Eq. (8) can come from the $SU(5)$ embedding without causing any conflict with the bounds on proton decay lifetime even if the leptoquark is very light, whereas the presence of the first term would require some fine tuning in order for the leptoquark not to couple to diquarks (see Appendix A). Note that if the first term is absent, then the singlet leptoquark cannot contribute to the D_s leptonic decay width. If the second term is absent the analysis is analogous to the triplet leptoquark scenario, with the exception that the singlet does not contribute to $K_L \rightarrow \mu^+ \mu^-$. Such is for example the case of the RPV minimal supersymmetric SM, where the interaction term of a right-handed down squark to quark and lepton doublets is present and corresponds to the first term in (8), while the second term is absent.

From the triplet scenario we know that $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ forces the Y_{1L}^ℓ couplings to be diagonal in the down-type quark basis and in particular $\tilde{Y}_{1L}^{d\ell} \approx 0$. Also relevant is the constraint from the lepton flavor universality ratio $Br(\tau \rightarrow K\nu)/Br(K \rightarrow \mu\nu)$ which receives relative leptoquark contributions of the form (9) with suitable flavor replacement ($c \rightarrow u$), while the tau semileptonic width is given as

$$\begin{aligned} \frac{\Gamma_{\tau \rightarrow K\nu}^{(1)}}{\Gamma_{\tau \rightarrow K\nu}^{SM}} &= \left| 1 + \frac{1}{4\sqrt{2}G_F} \right. \\ &\times \left. \left\{ \left[\frac{Y_{1L}^{u\tau*} \tilde{Y}_{1L}^{s\tau}}{V_{us}} \right] - \frac{m_\tau (Y_{1R}^{u\tau*} \tilde{Y}_{1L}^{s\tau})}{V_{us}^* (m_u + m_s)} \right\} \right|^2. \end{aligned}$$

Remaining constraint is the rare decay $D^0 \rightarrow \mu^+ \mu^-$ which in this case is of the form (7) with A and B

$$\begin{aligned} A &= \frac{m_D^2}{m_{\Delta_1}^4 (m_c + m_u)^2} |Y_{1R}^{u\mu} Y_{1L}^{c\mu*} - Y_{1R}^{c\mu*} Y_{1L}^{u\mu}|^2, \\ B &= \frac{1}{m_{\Delta_1}^4} \left| \frac{m_D}{m_c + m_u} (Y_{1R}^{u\mu} Y_{1L}^{c\mu*} + Y_{1R}^{c\mu*} Y_{1L}^{u\mu}) \right. \\ &\quad \left. - \frac{2m_\mu}{m_D} (Y_{1R}^{c\mu*} Y_{1R}^{u\mu} + Y_{1L}^{c\mu*} Y_{1L}^{u\mu}) \right|^2. \end{aligned}$$

The remaining relevant free parameters can correspondingly be chosen as an overall coupling magnitude δ and two angles (ϕ, ω) , defined through $\tilde{Y}_{1L}^{s\mu} = y_1^\mu \sin \omega$, $Y_{1R}^{c\mu} = y_1^\mu \cos \omega \cos \phi$ and $Y_{1R}^{u\mu} = y_1^\mu \cos \omega \sin \phi$. The value of y_1^μ is bounded from above by the condition of perturbativity ($y_1^\mu < \sqrt{4\pi}$). Together with existing direct experimental searches for second generation leptoquarks [26, 27] this gives an additional constraint on the possible size of the leptoquark contributions to the D_s leptonic width. By performing a numerical fit of (y_1^μ, ω, ϕ) to these constraints we obtain the result, that the experimental value for $Br(D_s \rightarrow \mu\nu)$ cannot be reproduced within one standard deviation without violating any of the other constraints, thus excluding the singlet leptoquark as a natural explanation of the $D_s \rightarrow \mu\nu$ puzzle.

As in the doublet case, the lack of experimental information on up-quark FCNCs involving only tau leptons leaves the verdict on the singlet leptoquark contribution to the $D_s \rightarrow \tau\nu$ decay width open. What is certain is that due to the $K^+ \rightarrow \pi^+\nu\bar{\nu}$ constraint any such contribution has to be aligned with the down-type quark Yukawas such that $\tilde{Y}_1^{d\tau} \approx 0$ can be ensured. In addition, the second term in Eq. (8) needs to be present and sizable to avoid the bounds coming from $Br(K \rightarrow \mu\nu)/Br(\tau \rightarrow K\nu)$.

VI. CONCLUSIONS

Scalar leptoquarks cannot naturally explain both enhanced $D_s \rightarrow \ell\nu$ decay widths due to existing constraints coming from precision kaon, tau, and D meson observables. The triplet leptoquark is excluded from contributing to any of the widths. Sizable contributions due to single right-handed down squark exchange in RPV supersymmetric models are also excluded, while a generic leptoquark singlet is definitely excluded only from explaining the $D_s \rightarrow \mu\nu$ width. The doublet contribution to this process is still technically allowed, while an

improvement in the search for $D^0 \rightarrow \mu^+\mu^-$ could very soon also completely exclude it. For the $D_s \rightarrow \tau\nu$ only the triplet (and RPV) explanations are already excluded, while the possible doublet explanation of both widths requires its mass to lie below 1.4 TeV and will certainly also be probed with direct leptoquark production at the LHC. Possible future signatures of a scenario where leptoquarks are responsible for the $D_s \rightarrow \tau\nu$ width could also be $Br(J/\psi \rightarrow \tau^+\tau^-)$ at the level of 10^{-11} , probably beyond the reach of BESIII [28], and also $Br(t \rightarrow c\tau^+\tau^-)$ at the level of 10^{-5} , close to the limiting sensitivity of the LHC [29].

Acknowledgments

This work is supported in part by the European Commission RTN network, Contract No. MRTN-CT-2006-035482 (FLAVIANet), the Marie Curie International Incoming Fellowship within the 6th European Community Framework Program (I.D.) and by the Slovenian Research Agency.

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 - [33] Phases of Y_3^{ij} could in principle conspire to yield a small contribution to $\Gamma_{\tau \rightarrow \eta\mu}$, however, such solutions cannot at the same time reproduce other related LFV decay limits, such as $\tau \rightarrow \eta'\mu$, $\tau \rightarrow \pi\mu$ or $\tau \rightarrow \phi\mu$.
 - [34] At this level of precision the main theoretical uncertainty in the SM comes from electromagnetic corrections, which have to be taken into account [31].
 - [35] A complementary triplet leptoquark contribution to the rare decay $D^0 \rightarrow \mu^+\mu^-$ presently gives a weaker constraint and is not included in Fig. 1

[36] Possible phases in Y_3^{ij} cannot invalidate this bound, since tau LFV decays require the $Y_3^{i\ell}$ couplings to different ℓ lepton flavors to be of different orders of magnitude.

Appendix A: $SU(5)$ EMBEDDING

We now demonstrate i) how natural it is for the weak triplet, doublet and singlet leptoquark interaction terms to arise in renormalizable $SU(5)$ model, and ii) how plausible it is for them to be light enough to play role in flavor physics phenomena.

In $SU(5)$, an i th ($i = 1, 2, 3$) generation of the SM matter fields comprises $\mathbf{10}_i = (\mathbf{1}, \mathbf{1}, 1)_i \oplus (\mathbf{3}, \mathbf{1}, -2/3)_i \oplus (\mathbf{3}, \mathbf{2}, 1/6)_i = (e_i^C, u_i^C, Q_i)$ and $\bar{\mathbf{5}}_i = (\mathbf{1}, \mathbf{2}, -1/2)_i \oplus (\bar{\mathbf{3}}, \mathbf{1}, 1/3)_i = (L_i, d_i^C)$, where $Q_i = (u_i \ d_i)^T$ and $L_i = (\nu_i \ e_i)^T$. The up quark (down quark and charged lepton) masses originate from the contraction of $\mathbf{10}_i$ and $\mathbf{10}_j$ ($\bar{\mathbf{5}}_j$) with 5- and/or 45-dimensional Higgs representation. (Observe that $\mathbf{10} \times \mathbf{10} = \bar{\mathbf{5}} \oplus \mathbf{45} \oplus \mathbf{50}$ and $\mathbf{10} \times \bar{\mathbf{5}} = \mathbf{5} \oplus \mathbf{45}$.) Only these two representations contain component that is both electrically neutral and an $SU(3)_c$ singlet that can thus obtain phenomenologically allowed vacuum expectation value (VEV). Actually, both are needed in a realistic renormalizable setting on purely phenomenological grounds.

The most general renormalizable set of Yukawa coupling contractions with $\mathbf{5}_H$ and $\mathbf{45}_H$ is

$$V = Y_{5^*}^{ij} \mathbf{10}_i^{\alpha\beta} \bar{\mathbf{5}}_{\alpha j} \mathbf{5}_{H\beta}^* + Y_5^{ij} \epsilon_{\alpha\beta\gamma\delta\epsilon} \mathbf{10}_i^{\alpha\beta} \mathbf{10}_j^{\gamma\delta} \mathbf{5}_H^\epsilon + Y_{45^*}^{ij} \mathbf{10}_i^{\alpha\beta} \bar{\mathbf{5}}_{\delta j} \mathbf{45}_{H\alpha\beta}^* + Y_{45}^{ij} \epsilon_{\alpha\beta\gamma\delta\epsilon} \mathbf{10}_i^{\alpha\beta} \mathbf{10}_j^{\zeta\gamma} \mathbf{45}_{H\zeta}^\epsilon,$$

where Greek indices are contracted in the $SU(5)$ space. Relevant fermion mass matrices are

$$M_D = (Y_{5^*}^T v_5^* + 2Y_{45^*}^T v_{45}^*) / \sqrt{2}, \quad (\text{A1a})$$

$$M_E = (Y_5^T v_5^* - 6Y_{45^*}^T v_{45}^*) / \sqrt{2}, \quad (\text{A1b})$$

$$M_U = [4(Y_5^T + Y_5) v_5 - 8(Y_{45}^T - Y_{45}) v_{45}] / \sqrt{2}, \quad (\text{A1c})$$

where $\langle \mathbf{5}_H^5 \rangle = v_5 / \sqrt{2}$, $\langle \mathbf{45}_{H1}^{15} \rangle = \langle \mathbf{45}_{H2}^{25} \rangle = \langle \mathbf{45}_{H3}^{35} \rangle = v_{45} / \sqrt{2}$ and $|v_5|^2 + |v_{45}|^2 = v^2$ ($v = 247 \text{ GeV}$). Y_{5^*} , Y_{45^*} , Y_5 and Y_{45} are arbitrary 3×3 Yukawa matrices.

If only 5-dimensional (45-dimensional) Higgs representation were present one would have $M_E^T = (-3)M_D$. A scenario with only one Higgs representation would hence yield $m_\tau/m_b = m_\mu/m_s = m_e/m_d$ at the GUT scale, which is in conflict with what is inferred from experimental observations. This is why both $\mathbf{5}_H$ and $\mathbf{45}_H$ are needed at renormalizable level. (Note, since $m_\mu/m_s > 1$ whereas $m_\tau/m_b \approx 1$ at the GUT scale this would suggest that the $Y_{45^*}^{22}$ entry is enhanced compared to other entries of Y_{45^*} [30].)

Conveniently enough, the 45-dimensional Higgs representation $\mathbf{45}_H = (\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7) =$

$(\mathbf{8}, \mathbf{2}, 1/2) \oplus (\bar{\mathbf{6}}, \mathbf{1}, -1/3) \oplus (\mathbf{3}, \mathbf{3}, -1/3) \oplus (\bar{\mathbf{3}}, \mathbf{2}, -7/6) \oplus (\mathbf{3}, \mathbf{1}, -1/3) \oplus (\bar{\mathbf{3}}, \mathbf{1}, 4/3) \oplus (\mathbf{1}, \mathbf{2}, 1/2)$ contains a weak triplet (Δ_3), doublet (Δ_4), and singlet (Δ_5) leptoquarks we are interested in whereas the 5-dimensional Higgs representation, $\mathbf{5}_H$, contains a singlet leptoquark only. So, these leptoquark states must be present in any renormalizable theory based on $SU(5)$.

The most stringent constraints on leptoquark masses and their couplings to matter originate from limits on partial proton decay lifetimes. In that respect only Δ_4 is innocuous enough since it does not directly mediate proton decay. (It cannot couple to a quark-quark pair.) It is also practically impossible for it to be a part of the process that destabilizes proton through mixing with the Higgs doublet and some other state that couples to a quark-quark pair since $(\bar{\mathbf{3}}, \mathbf{2}, 1/6)$ – the only suitable candidate – is not part of either 5- or 45-dimensional Higgs representation (or 24-dimensional representation). It is thus phenomenologically possible for Δ_4 to be light and have couplings to the matter fields of the form given in Eq. (5) in $SU(5)$. In that case $Y_{2L} = -2^{1/2}[Y_{45}^\dagger - Y_{45}^*]$ and $Y_{2R} = Y_{45^*}^\dagger$.

It is also possible to have Δ_3 that couples to the quark-lepton pairs and no proton decay. In particular, the $\mathbf{10}\text{-}\bar{\mathbf{5}}\text{-}\mathbf{45}_H^*$ contraction yields a lepton-quark pair couplings with Δ_3 of the form given in Eq. (2): $Y_3 = Y_{45^*}^\dagger$. On the other hand, the $\mathbf{10}\text{-}\mathbf{10}\text{-}\mathbf{45}_H$ contraction yields couplings of Δ_3 to a quark-quark pair only. Clearly, if only one of these two possible contractions is present there would not be a tree level proton decay due to Δ_3 . In the former case there would not be proton decay due to the mixing of Δ_3 with the Higgs doublet and some other states either since $(\mathbf{3}, \mathbf{1}, 2/3)$ – which would be a suitable candidate – is not part of either 5- or 45-dimensional representation.

Finally, Δ_5 could also be coupled to matter in a manner that renders proton stable contrary to the usual expectation. Namely, the $\mathbf{10}\text{-}\mathbf{10}\text{-}\mathbf{45}_H$ contraction yields couplings to a lepton-quark pair only. (This should be compared to the $\mathbf{10}\text{-}\mathbf{10}\text{-}\mathbf{5}_H$ contraction that generates both the lepton-quark and quark-quark type of couplings simultaneously for the singlet leptoquark in $\mathbf{5}_H$.) This contraction yields the second term in Eq. (8): $Y_{1R} = 2^{1/2}[Y_{45}^\dagger - Y_{45}^*]$. The $\mathbf{10}\text{-}\bar{\mathbf{5}}\text{-}\mathbf{45}_H^*$ contraction, on the other hand, yields not only the second term in Eq. (5), i.e., $Y_{1L} = -2^{1/2}Y_{45^*}^\dagger$, but also the quark-quark couplings which would lead to proton instability. If only $\mathbf{10}\text{-}\mathbf{10}\text{-}\mathbf{45}_H$ contraction is present proton could be stable and accordingly Δ_5 could be light. Interestingly enough, it is possible to have a scenario in which there would not be any leptoquark induced proton decay. The necessary condition for this to happen would be the absence of the $\mathbf{10}\text{-}\bar{\mathbf{5}}\text{-}\mathbf{45}_H^*$ contraction.